APPENDIX D

RELIABILITY ANALYSIS OF A SIMPLE SLOPE

D-1. Introduction.

a. The reliability analysis of a simple slope stability problem is presented and is intended to demonstrate the methodology of geotechnical reliability analysis using the Corps of Engineers reliability index method of analysis. The guidance for performing geotechnical reliability analysis is given in ETL 1110-2-547.

b. When performing reliability analyses, the same methods of analysis are used as are used in traditional geotechnical engineering. The only difference is in how the variables in the problem are represented.

c. In reliability analysis there are two types of variables: deterministic and random variables.

(1) Deterministic variables are represented by a single value because the value of that variable is known exactly. A deterministic variable could be the unit weight of water or measured dimensions.

(2) Random variables are represented by a probability density function, which defines the relative likelihood that the random variable assumes various ranges of values. Random variables could be shear strength, permeability, and earth pressure.

d. A probability density function is a plot of the value of the random variable on the x-axis and the likelihood of occurrence of that value on the y-axis. An important property of a probability density function is that the area under the curve is equal to one.

e. The field of statistics has defined the first four characteristics or moments of each probability density function as the mean (expected value), the variance, the coefficient of skewness, and the coefficient of kurtosis. The Corps of Engineers reliability index method of analysis uses what is called a first order second moment method of reliability analysis. This means that only the first term of the Taylor series expansion about the mean value (first order) is used and the first two moments (mean and variance) are used to represent the probability density function in the reliability analysis. This first order second moment method of analysis greatly simplifies the reliability analysis procedure.

D-2. Reliability Analysis Terms and Basic Equations.

a. The first two moments of the probability density function are the mean and variance. In engineering terms, the mean is the centroid of the probability density function and the variance is the central moment of inertia.

b. The mean μ_x is represented by Equation 1:

$$\mu_x = \sum \frac{x_i}{N} \tag{1}$$

where x_i is a data point and N is the number of data points.

c. In reliability analysis the mean is also referred to as the expected value *E(x)*.

d. The variance *Var[x]* is represented by Equation 2:

$$Var[x] = \frac{\sum [(x_i - \mu_x)^2]}{N-1} \tag{2}$$

e. The standard deviation σ_x is represented by Equation 3:

$$\sigma_x = \sqrt{Var(x)} \tag{3}$$

f. Inversely, *Var(x)* is equal to the square of the standard deviation of *x*. The variance and standard deviation are measures of the dispersion or variability of the random variable. The larger the standard deviation the larger the range of values the random variable may assume. The smaller the standard deviation the smaller the range of values the random variable may assume. As the value of the standard deviation gets smaller and smaller the random variable will approach a constant or a deterministic variable in the limit.

g. The expected value of *x* will be used to represent the first moment, and the standard deviation of *x* will be used to represent the second moment of the random variable.

h. The coefficient of variation V_x is represented by Equation 4:

$$V_x = \frac{\sigma_x}{E[x]} \times 100\% \tag{4}$$

i. As some of the variables in a geotechnical analysis are represented as random variables, i.e., a probability density function, it only stands to reason that the results of the analysis will be a probability density function. The results of many geotechnical analyses are expressed in terms of factors of safety (*FS*). In traditional geotechnical analyses, the *FS* is a single number. In reliability analyses, the *FS* is a random variable represented by a distribution (probability density function), not a single number.

j. The Corps of Engineers has chosen to represent the probability density function of the *FS* as a lognormal probability density function (see Figure D-1). The lognormal probability density function is shown at the bottom of Figure D-1. The transformed normal probability density function is shown at the top of Figure D-1. The lognormal probability density function is transformed to the normal probability density function by taking the natural log of the factor of safety. One reason for choosing the probability density function of the *FS* as a lognormal probability density function is that a lognormal function can never have a value less than zero. Thus, the FS will not have negative values. Another reason is that when one takes the natural log of the *FS*, the distribution of the *FS* becomes a normal distribution. A third reason is that a probability density function resulting from the product of many variables is lognormally distributed. In geotechnical analyses typically many numbers are multiplied together. For the lognormal distribution of the *FS* shown in Figure D-1, the hatched area under the curve and to the left of a *FS* = 1 gives the probability that the *FS* is less than one or the probability that the performance of the slope stability problem will not be satisfactory. *P(u)* is defined as the probability of unsatisfactory performance, i.e., the probability that the *FS* is less than 1. When the lognormal distribution is transformed to the normal distribution by taking the natural log of the *FS* it is noted that the ln(1) = 0. Thus the *P(u)* is represented by the hatched area under the normal distribution curve to the left of zero.

k. On the normal distribution curve in Figure D-1 the distance between zero and *E[ln(FS)]* is defined as $\beta\sigma_{ln(FS)}$. Beta (β) is the reliability index. The reliability index is a measure of the distance that the expected value of the FS is away from unsatisfactory performance. The larger the value of β, the more reliable the slope is. The smaller the value of β, the closer the slope condition is to unsatisfactory performance. Calculated values of β greater than three represent a stable slope whereas values of β less than 2 represent a poor performing slope. Values of β less than 1.5 represent unsatisfactory performance.

l. The reliability index β is represented by Equation 5:

$$\beta_{\log normal} = \frac{\ln\left[\dfrac{E(FS)}{\sqrt{1+V_{FS}^2}}\right]}{\sqrt{\ln(1+V_{FS}^2)}} \tag{5}$$

m. Once β is calculated, the probability of unsatisfactory performance can be calculated from Equation 6:

$$P(u) = \Phi(-\beta) \tag{6}$$

where $\Phi(-\beta)$ is obtained from a standard normal probability density function N(0,1). A standard normal probability density function has a mean of zero and a standard distribution of one. Table D-1 gives the area under the standard normal probability density function for values of *x* where *x* is given by Equation 7:

$$x = -\beta \tag{7}$$

n. For values of *x* to one decimal point the area under the standard normal probability distribution to the left of the *x* line (the shaded area) can be read from the table to the right of the *x* value in the first column. The second decimal of *x* is given across the top of the table. For example, the area under the standard normal probability distribution function to the left of *x* for a value of *x* = 1.55 is 0.9394. Note this area represents the probability of unsatisfactory performance for a beta of -1.55.

o. The reliability is the probability that the embankment slope will not experience unsatisfactory performance or that the slope will perform in a stable or reliable manner. The reliability R is represented by Equation 8:

$$R = 1 - P(u) \tag{8}$$

D-3. Taylor Series Method of Reliability Analysis.

a. The Taylor series method of reliability analysis will be used to compute the reliability index. This method is based on a Taylor series expansion of the performance function about some point. For this analysis the expansion is performed about the expected values of the random variables. Since this is a first order method, only the first order (linear) terms of the Taylor series expansion are retained. Equation 9 is obtained from the Taylor series expansion using the assumption that the first derivatives (slopes) are calculated at the expected value using an interval of plus or minus one standard deviation of the random variable.

$$Var(FS) = \sigma_{FS}^2 = \sum_{i=0}^{n} \left[\frac{FS(x_i + \sigma_{x_i}) - FS(x_i - \sigma_{x_i})}{2} \right]^2 \tag{9}$$

b. Equation 9 gives the variance of the factor of safety *(Var(FS))* where *x* is the random variable and there are *n* random variables. The Taylor series method of reliability analysis requires 2n+1 geotechnical analyses (slope stability analyses) to be performed. A benefit of using the Taylor series method of analyses is that the relative contribution of each random variable to the total uncertainty can be determined.

D-4. Determining Expected Values and Standard Deviation.

a. The advantage of using a first order second moment method of reliability analysis is that the probability density function of each random variable can be represented by only two values, the expected value and the standard deviation. The expected value of a random variable is fairly easy to obtain. There are a number of sources from which the expected values of soil properties can be estimated or obtained: testing (laboratory or in-situ), other projects in the area, back calculation of the value, experience, manuals and tables, or correlation with soil index properties.

b. Obtaining the expected value is the easy part, that is, using laboratory data, historical data, estimates based on field exploration and experience. But geotechnical engineers generally do not have a feel for the value of the standard deviation of soil properties. The standard deviation of soil properties is more difficult to obtain but there are three methods to obtain it.

(1) Data. If there is a substantial amount of test data for the soil properties, the expected value and the standard deviation can be calculated using Equations 1, 2, and 3. This is the best method to use. But a substantial amount of test data may not exist.

(2) Coefficient of Variation Tables. In the absence of test data values of coefficients of variation for soil properties can be found in the literature. Table D-2 gives coefficients of variation found in the literature or calculated from those values and the reference. From the expected value and the coefficient of variation, the standard deviation can be calculated from Equation 4.

(3) Six Sigma Rule. For random variables for which a coefficient of variation cannot be found in the literature, the six-sigma rule can be used. The six-sigma rule makes use of the experience of geotechnical engineers. The six-sigma rule is given by Equation 10.

$$\sigma = \frac{Largest\ Possible\ Value - Smallest\ \ Possible\ \ Value}{6} \tag{10}$$

(a) The six sigma rule is often called the three-sigma rule because it determines the value of the parameter at plus and minus three standard deviations from the mean or expected value. See Appendix C for more details on the six sigma rule.

(b) By examining the normal probability density function in Figure D-2, the justification for the six sigma rule can be demonstrated. Figure D-2 shows a normal probability density function with the percentage of the area under the curve for the mean plus or minus one standard deviation, two standard deviations, and three standard deviations (six sigma). For the mean plus or minus three standard deviations, 99.73% of the area under the normal distribution is included. So for a range of six sigma (the highest possible value to the lowest possible value), essentially all the values represented by the normal distribution curve are included, thus the name the six sigma rule. The shear strength of sand (ϕ) can be used to demonstrate the six sigma rule. The highest possible value of ϕ for a sand is 45°. The lowest possible value of ϕ is 25°. Applying the six sigma rule results in the following:

$\sigma = (45°-25°)/6 = 3.33°$

(c) For a sand with an expected value for ϕ of 35°, using the coefficient of variation Table D-2, sigma is calculated using Equation 4 as

$\sigma = 0.10(35°) = 3.5°$

where 10% for the coefficient of variation (V_x) comes from Table D-2. The value of sigma of 3.33 from the six sigma rule compares well with the value of sigma of 3.5 from the coefficient of variation method.

D-5. Example Problem – Infinite Slope Reliability Analysis.

a. A simple slope stability problem can be examined to see how the reliability analysis presented above is performed. A simple embankment is shown in Figure D- 3. The embankment is constructed of sand. The embankment slope is shown as approximately 1 vertical on 1.5 horizontal. The goal is to determine how stable this slope is, that is, the probability of unsatisfactory performance of the embankment. As was stated before, the same methods of analysis will be used for the reliability analysis as are used in a traditional geotechnical slope stability analysis. There are several methods of analysis that can be used: force equilibrium slope stability computer programs, slope stability chart solutions, and the infinite slope equation. To simplify the reliability analysis, the infinite slope equation given by Equation 11 will be used.

$$FS = b \tan \phi \qquad (11)$$

where FS is the factor of safety, b defines the slope of the embankment when the slope is 1 vertical on b horizontal, and ϕ is the angle of internal friction of the sand. There are two random variables in this reliability analysis, ϕ and b. The expected value and the standard deviation need to be determined for each random variable. Based on previous work at this site the expected value of the angle of internal friction of the sand ($E[\phi]$) is 38°. The standard deviation can be obtained from Equation 4 and Table D- 2. From Table D-2 the coefficient of variation of the angle of internal friction for sand is 10%. The standard deviation is calculated as

$\sigma_\phi = 0.1 \times 38° = 3.8°$

b. The expected value of b ($E[b]$) is 1.5. The six-sigma rule will be used to determine the standard deviation of b. From looking at this embankment the geotechnical engineer has determined that the embankment slope varies between a maximum value of b = 1.625 and a minimum value of b = 1.375. The standard deviation of b (σ_b) is calculated as follows using Equation 10

$\sigma_b = (1.625 – 1.375)/6 = 0.042$

c. Since a first order second moment method of analysis is being used, the two random variables are defined as follows:

Sand	$E[\phi] = 38°$	$\sigma_\phi = 3.8°$
Slope	$E[b] = 1.5$	$\sigma_b = 0.042$

d. The Taylor series method of reliability analysis will be used. Since there are two random variables, 2n+1 calculations will need to be accomplished.

2n+1 = 2 x 2 + 1 = 5

e. The first calculation will be to determine the expected value of the factor of safety (*E[FS]*). Using the infinite slope equation (11), we get the following

$E[FS] = E[b]\tan(E[\phi]) = 1.5 \tan 38 = 1.17$

f. Next determine the expected values of the random variables at plus and minus one standard deviation.

$E[\phi]+\sigma_\phi = 38 + 3.8 = 41.8$

$E[\phi]-\sigma_\phi = 38 - 3.8 = 34.2$

$E[b]+\sigma_b = 1.5 + 0.042 = 1.542$

$E[b]-\sigma_b = 1.5 - 0.042 = 1.458$

g. Using these values, the infinite slope equation is used to calculate the factor of safety for ϕ plus and minus one standard deviation while holding *b* constant and for *b* plus and minus one standard deviation while holding ϕ constant.

$FS_{\phi+\sigma} = 1.5 \tan(41.8) = 1.34$

$FS_{\phi-\sigma} = 1.5 \tan(34.2) = 1.02$

$FS_{b+\sigma} = 1.542 \tan(38) = 1.20$

$FS_{b-\sigma} = 1.458 \tan(38) = 1.14$

h. The results of these calculations are presented in Table D- 3.

Table D-3. Summary of Taylor Series Method Reliability Calculations

Random Variable	ϕ	B	FS	Var(FS)	Percent	Remarks
$E[\phi]$, $E[b]$	38	1.5	1.17			
$E[\phi]+\sigma_\phi$, $E[b]$	41.8	1.5	1.34	0.026	96.7	Variable that controls analysis
$E[\phi]-\sigma_\phi$, $E[b]$	34.2	1.5	1.02			
$E[\phi]$, $E[b]+\sigma_b$	38	1.542	1.20	0.0009	3.3	Little effect on analysis
$E[\phi]$, $E[b]-\sigma_b$	38	1.458	1.14			
			$\sum$	0.0269	100	

i. Instead of using the infinite slope equation to calculate the factor of safety in each of the above five slope stability analyses, other methods of slope stability analysis could have been used: force equilibrium slope stability computer programs or slope stability chart solutions. No matter what method of analysis is used the end result is that 2n+1 factors of safety are calculated.

j. The next step in the reliability analysis is to use Equation 9 to calculate the variance of the factor of safety (*Var(FS)*) for each random variable. The variance of the factor of safety for the random variable ϕ is calculated using the factor of safety for ϕ plus one standard deviation minus the factor of safety for ϕ minus one standard deviation divided by two and the quantity squared.

$\mathrm{Var(FS)}_{\phi} = [(1.34 - 1.02)/2]^2 = 0.026$

k. The variance of the factor of safety for the random variable *b* is calculated in the same manner.

$\mathrm{Var(FS)}_{b} = [(1.20 - 1.14)/2]^2 = 0.0009$

l. The variance of the factor of safety (*Var(FS)*) is the sum of the variances for the factors of safety of all random variables.

Var(FS) = 0.026 + 0.0009 = 0.0269

m. One big advantage of using the Taylor series method of analysis is that the random variable that controls the reliability analysis can easily be determined. The variance of the factor of safety for each random variable divided by the sum of all the variances gives the percent that that random variable effects the analysis. In Table D- 3 under the column labeled "percent" we see that the random variable ϕ controls 96.7% of the analysis and that the random variable *b* controls 3.3% of the analysis. This indicates that a similar result would have been obtained if *b* had been chosen to be a deterministic variable.

n. Using Equation 3 the standard deviation of the factor of safety can be calculated from the variance of the factor of safety as follows:

$\sigma_{FS} = (0.0269)^{0.5} = 0.16$

o. The coefficient of variation of the factor of safety (V_{FS}) can then be calculated from the expected value of the factor of safety (*E[FS]*) and the standard deviation of the factor of safety (σ_{FS}) as follows using Equation 4.

$V_{FS} = 0.16/1.17 = 0.14$

p. Using the values of the expected value of the factor of safety and the coefficient of variation of the factor of safety, the value of $\beta_{lognormal}$ can be calculated using Equation 5.

$$\beta_{\log normal} = \frac{\ln\left[\frac{1.17}{\sqrt{1+0.14^2}}\right]}{\sqrt{\ln(1+0.14^2)}} = 1.06$$

For the embankment shown in Figure D-3 the stability of the slope is calculated to have a value of beta equal to 1.06.

q. The probability of unsatisfactory performance can be calculated using Equations 6 and 7 and Table D-1 as follows. Table D-1 gives the area under the standard normal distribution curve for a value of *x*. In this case *x* = -1.06 (from Equation 7). The area under the curve in Table D-1 for *x* = -1.06 is 0.1446. This area is the probability of unsatisfactory performance.

P(u) = Φ(-β) = Φ(-1.06) = 0.1446

which is rounded to two significant decimal places to

P(u) = 0.14

Note that previously it was indicated that values of beta less that 1.5 represent unsatisfactory performance.

r. One way to comprehend or communicate 14% probability of unsatisfactory performance is the concept that of 100 embankments constructed exactly like this one, 14 of them would have unsatisfactory performance. This probability of unsatisfactory performance is then used with the other parameters from the event tree to calculate risk.

D-6. <u>When the FS is Greater Than 1.</u>

a. When unsatisfactory performance is associated with FS>1, the associated probability cannot be determined from the reliability index β. A more general index can be derived from the definition of the normalized parameter *z*,

$$z(FS_u) = \frac{E[\ln(FS)] - \ln(FS_u)}{\sigma_{\ln(FS)}} \tag{12}$$

b. The probability of unsatisfactory performance is $P(u) = P(FS<FS_u) = \beta(-z)$. Using the relationships between the mean and variance of the factor of safety and the parameters for the log normal distribution as given by

$$E[\ln(FS)] = \ln(E[FS]) - \frac{\sigma^2_{\ln(FS)}}{2} \tag{13}$$

and

$$\sigma_{\ln(FS)} = \sqrt{\ln(1+V_{FS}^2)} \tag{14}$$

where,

$$V_{FS} = \frac{\sigma_{FS}}{E[FS]}, \tag{15}$$

$z(FS_u)$ is found to be

$$z(FS_u) = \frac{\ln\left(\frac{E[FS]/FS_u}{\sqrt{1+V_{FS}^2}}\right)}{\sqrt{\ln(1+V_{FS}^2)}}. \tag{16}$$

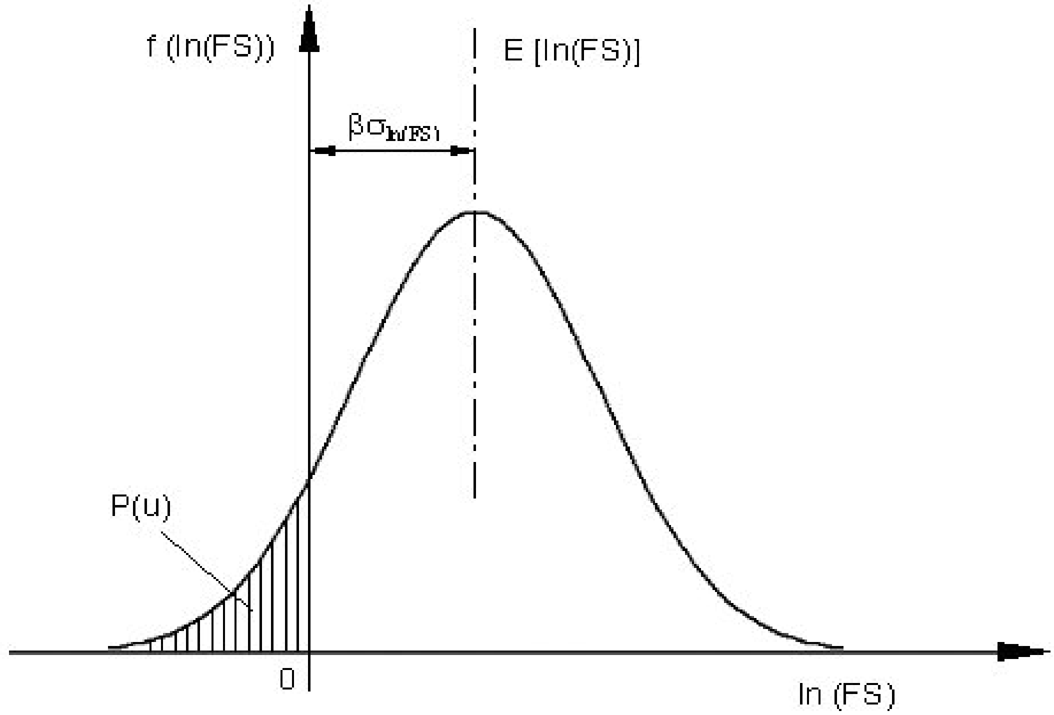
c. Consider a case where experience show that excessive deformation occurs when FS<1.1. Using the data from the previous example

$$z(FS_u) = \frac{\ln\left[\frac{1.17/1.1}{\sqrt{1+0.14^2}}\right]}{\sqrt{\ln(1+0.14^2)}} = 0.37$$

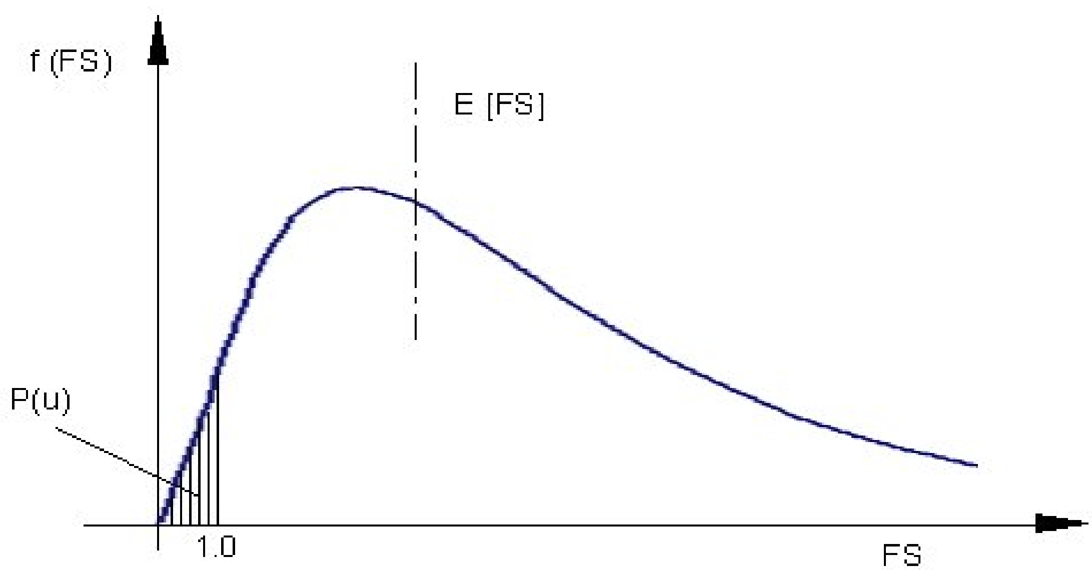
The probability of unsatisfactory performance is therefore

$P(FS<FS_u) = \Phi(-0.37) = 0.36$

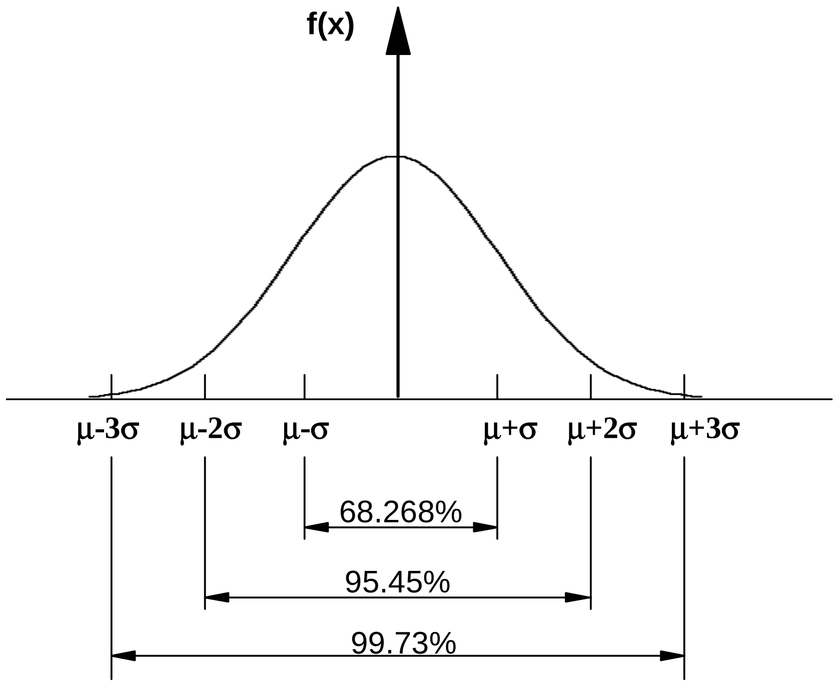
d. Note that this probability includes both unsatisfactory performance by slope failure and unsatisfactory performance by excessive deformation. If these two risks imply different consequences, the probabilities would have to be separated. In this case the probabilities of unsatisfactory performance by failure is a subset of unsatisfactory performance by excessive deformation. Therefore, the probability for slope failure is 0.14 while the probability for unsatisfactory performance by excessive deformation excluding slope failure is 0.36-0.14=0.22.



Transformed Probability Density Function of Factor of Safety



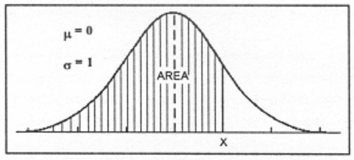
Lognormal Probability Density Function of Factor of Safety
Figure D-1



Normal Densitv Function
Figure D - 2

Embankment
Figure D - 3

Table D-1. Area Under the Standard Normal Probability Density Function



x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table D-2
COEFFICIENT OF VARIATION

Parameter	Coefficient of Variation (%)	Reference
Unit Weight	3	Harr (1987)
Coefficient of Permeability (k)	90	Harr (1987)
Angle of Internal Friction (Sand) (ϕ)	10	LD25 Rehab Report (1992)
Cohesion (Undrained shear strength) (c)	40	Harr (1987)
Wall Friction Angle (δ)	20	S&W Report (1994)
Wall Friction Angle (δ)	25	ETL 1110-2-354
Earth Pressure Coefficient Sand (K)	15	Calculated
Downdrag Coefficient Sand (K_v)	25	Calculated
Standard Penetration Test	26	Harr (1987)
Standard Cone Test	37	Harr (1987)
Lag Factor (Silt and Sand Levees)	20	Clough (1966)
Dead Load	10	Harr (1987)
Live Load	25	Harr (1987)
Wind Load	37	Harr (1987)
Impact Load	30	ETL 1110-2-354
Hawser Pull Load	20	ETL 1110-2-354
Seismic Force	30	ETL 1110-2-354
Constant of Horizontal Subgrade Reaction (n_h)	25	ETL 1110-2-354
Group Reduction Factor (R_g)	8	ETL 1110-2-354

Table D-2
COEFFICIENT OF VARIATION

Parameter	Coefficient of Variation (%)	Reference
Cyclic Reduction Factor (R_c)	25	ETL 1110-2-354
Timber Piles, Sand Foundation		
Ultimate Compression Capacity	25	
$\phi = 35°$	22	Calculated
$\phi = 40°$	38	Calculated
Ultimate Tension Capacity	18	Calculated
Pile Diameter	8	Historic Data